Elastic Strings and Springs

$Hooke's\ Law$:

$$T \propto x$$

$$T = kx$$

$k\ is\ the\ stiffness\ constant\ n \cdot m^{-1}$

$$\boxed{T = \frac{\lambda x}{l}}$$

$\lambda = Young's\ Modulus$

$l = natural\ length$

Elastic Potential Energy

$$NSL\ \ F = ma$$

$$F = mv\frac{dv}{dx}$$

$$\int_{x_1}^{x_2} F\,dx = \int_{x_1}^{x_2} mv\frac{dv}{dx}\,dx$$

$$\int_{x_1}^{x_2} F\,dx = \int_{v_1}^{v_2} mv\,dv$$

$$= \left[\frac{1}{2}mv^2\right]_{v_1}^{v_2}$$

$$= \frac{1}{2}m{v_2}^2 - \frac{1}{2}m{v_1}^2$$

$$= \Delta E_k = Work\ done$$

$$\therefore Work\ done = \int_{x_1}^{x_2} F\,dx$$

$$Hooke's\ Law\ \ F = kx$$

$$Work\ done = \int_{x_1}^{x_2} kx\, dx$$

Work done in stretching the string from its natural length to an extension of x

$$\int_0^x kx\, dx = \frac{1}{2}kx^2 = \frac{\lambda x^2}{2l}$$

$$Elastic\ Potential\ Energy$$

$$= \frac{\lambda x^2}{2l}$$

Motion in a Vertical Circle

$$Angular\ Velocity\ \omega = \frac{d\theta}{dt} = \dot{\theta}$$

$$Transverse\ Velocity\ v = \frac{dx}{dt} = \frac{r\theta}{\Delta t} = r\omega$$

$$Angular\ Acceleration\ \alpha = \frac{d\omega}{dt} = \dot{\omega} = \ddot{\theta}$$

<u>Acceleration is the resultant of two components:</u>

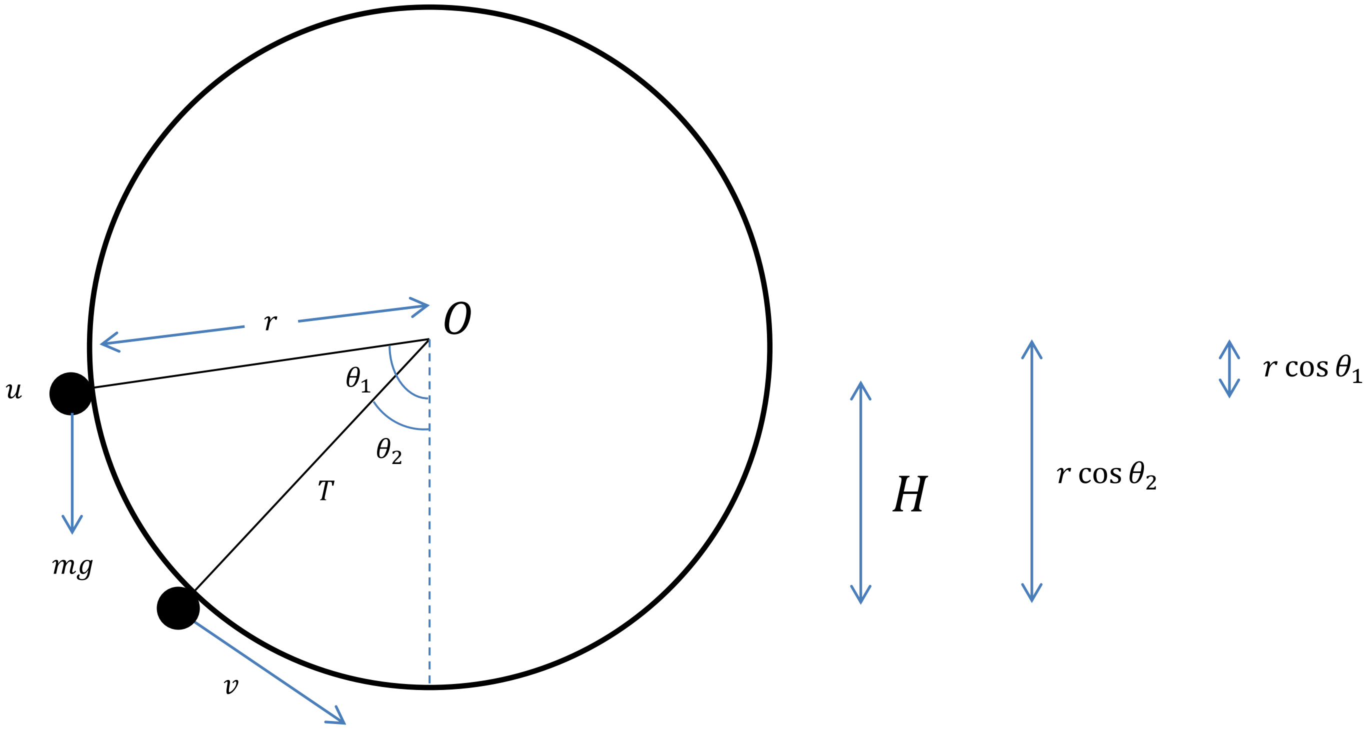
$$Radial\ Component\ of\ a = r\omega^2 = \frac{v^2}{r}$$

$Transverse\ Component\ of\ a$:

$$F = ma$$

$$mg\sin\theta = ma$$

$$\boxed{a = g\sin\theta}$$



$$H = r\cos\theta_2 - r\cos\theta_1 = r(\cos\theta_2 - \cos\theta_1)$$

$$E_u = \frac{1}{2}mu^2$$

$$E_v = \frac{1}{2}mv^2 - mgH = \frac{1}{2}mv^2 - mgr(\cos\theta_2 - \cos\theta_1)$$

$By\ conservation\ of\ energy$:

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 - mgr(\cos\theta_2 - \cos\theta_1)$$

$$u^2 = v^2 - 2gr(\cos\theta_2 - \cos\theta_1)$$

$$\boxed{v^2 = u^2 + 2gr(\cos\theta_2 - \cos\theta_1)}$$

Tension around a Circle

$$a = \frac{v^2}{r} \quad F = ma$$

$$F = \frac{mv^2}{r}$$

$$T - mgcos\theta = \frac{mv^2}{r}$$

$$T = mgcos\theta + \frac{mv^2}{r}$$

$where\ mgcos\theta = radial\ component\ of\ weight$

$String\ goes\ slack\ when\ tension\ T = 0$

Partial Fractions

$$\frac{f(x)}{g(x)} = \frac{f(x)}{P(x) \cdot Q(x)}$$

$$\frac{f(x)}{P(x) \cdot Q(x)} = \frac{A}{P(x)} + \frac{B}{Q(x)}$$

$$multiply\ by\ g(x)$$

$$f(x) = A \cdot Q(x) + B \cdot P(X)$$

Solve for A and B by letting x equal the roots of g(x)

Integration by Substitution

$$I = \int \frac{dx}{a^2 + x^2}$$

$$let\ x = atan\theta$$

$$\therefore dx = a \sec^2 \theta \, d\theta$$

$$and\ \theta = \tan^{-1} \frac{x}{a}$$

$$I = \int \frac{a \sec^2 \theta}{a^2 + a^2 \tan^2 \theta} d\theta$$

$$= \int \frac{a\sec^2 \theta}{a^2(1 + \tan^2 \theta)} d\theta$$

$$= \frac{1}{a} \int 1 \, d\theta$$

$$= \frac{1}{a} \theta + k$$

$$\boxed{\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + k}$$

Impulse and Momentum

$$Impulse = Change\ in\ momentum$$

$$I = \Delta p$$

$$\boxed{I = m(v - u)}$$

$$NSL\ \ F = ma$$

$$F = m\frac{dv}{dt}$$

$$\int F\,dt = \int m\frac{dv}{dt}\,dt$$

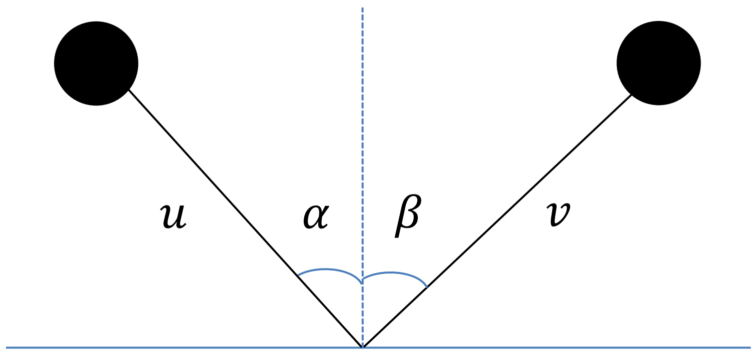
$$\int_{t_1}^{t_2} F\,dt = \int_u^v m\,dv = [mv]_u^v$$

$$= mv - mu = \Delta p$$

$$\boxed{v = eu}$$

$$where\ e = coefficient\ of\ restitution$$

$$\boxed{v_1 - v_2 = -e(u_1 = u_2)}$$



Impulse Perpendicular to Plane

$$\boxed{I_{\perp} = m(v\cos\beta + u\cos\alpha)}$$

Impulse Parallel to Plane

$$I_{\parallel}:\ mu\sin\alpha = mv\sin\beta$$

$$\boxed{u\sin\alpha = v\sin\beta}$$

Restitution:

$$\boxed{eu\cos\alpha = v\cos\beta}$$

$$\frac{u\sin\alpha}{eu\cos\alpha} = \frac{v\sin\beta}{v\cos\beta}$$

$$\tan\alpha = e\tan\beta$$

$$or\ e\cot\alpha = \cot\beta$$